Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT-PH : Mathematics for Physics – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Define the gradient field of a differentiable function f(x, y, z).
2. Define the circulation density of a vector field F = Mi + Nj at the point (x, y).
3. Give an example for a non-orientable surface.
4. State the Trapezoidal rule for Numerical Integration.
5. Find the order of the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x}$. **(4×1=4)**

PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

6. Find the curl of $F = (x^2 - z)i + xe^z j + xyk$.
7. State Stoke's theorem for a smooth oriented surface.
8. With the usual notations, prove that $\nabla \times \nabla f = 0$.
9. The vector field F(x, y, z) = xi + yj + zk represent the velocity of a gas flowing in space. Show that the gas is undergoing constant uniform expansion at all points.

P.T.O.

10. Find a parametrization of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$.

11. Evaluate the line integral $\int_C xy\,dy - y^2 dx$ where C is the square cut from the first quadrant by the lines x = 1 and y = 1.

12. Evaluate $\int_{-3}^{3} x^4\,dx$ by using Simpson's 1/3 rule.

13. Evaluate $\int_0^6 \frac{1}{1+x}\,dx$ using Trapezoidal rule.

14. Describe the fourth order Runge-Kutta formula.

15. Show that $u = x^2 - y^2$ is a solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

16. Solve $u_{xy} = -u_x$. **(7×2=14)**

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

17. Find the work done by the conservative field F = yzi + xzj + xyk along any smooth curve C joining the point A(– 1, 3, 9) to B(1, 6, – 4).

18. Find the flux of F = (x – y)i + xj across the circle $x^2 + y^2 = 1$ in the xy-plane.

19. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).

20. Apply Simpson's one third rule to evaluate $\int_1^6 \frac{1}{1+x^2}\,dx$ with h = 1.

21. From the Taylor series for y(x), find y(0.1) correct to four decimal places if y(x) satisfies $y' = x - y^2$, y(0) = 1.

22. Solve the wave equation $u_{tt} - c^2 u_{xx} = 0$.

23. If u_1 and u_2 are solutions of $u_t = c^2 u_{xx}$ in some region R. Prove that $u = c_1 u_1 + c_2 u_2$ is also a solution of the above partial differential equation. **(4×3=12)**

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

24. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9$, $0 \leq z \leq 5$.

25. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

26. Given : $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$. Find $y(0.2)$ and $y(0.4)$.

27. Find the type, transform to normal form, and solve the partial differential equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$. **(2×5=10)**