K21U 6799

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2021
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT – PH : Mathematics for Physics – I

Time : 3 Hours

Max. Marks : 40

PART – A

Answer **any 4** questions from among the questions 1 to 5. **Each** question carries **1** mark.

1. If $y = \sin^{-1} x$, show that $(1 - x^2)y_2 - xy_1 = 0$.

2. Verify Rolle's theorem for the function $f(x) = e^x (\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

3. Determine the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 3 \\ 2 & 1 & 5 \end{bmatrix}$.

4. Write the polar equation of the circle $x^2 + (y - 2)^2 = 4$.

5. Find $\frac{dy}{dx}$ if $ay^2 = x^3$.

PART – B

Answer **any 7** questions from among the questions 6 to 15. **Each** question carries **2** marks.

6. Find the n^{th} derivative of $\frac{x+3}{(x-1)(x-2)}$.

7. If $x = \frac{1}{2}\left(t - \frac{1}{t}\right)$, $y = \frac{1}{2}\left(t + \frac{1}{t}\right)$, find $\frac{d^2y}{dx^2}$.

8. If $y = e^{5x}$ sin3x, prove that $y_2 - 10y_1 + 34y = 0$.

9. Write the Maclaurin's series expansion of tan x with at least three terms with non zero coefficients.

10. If x is positive, prove that $\log(1+x) \geq x - \frac{x^2}{2}$.

11. Expand $\tan^{-1}x$ in powers of $x - 1$.

12. Verify Cauchy's Mean Value Theorem for the function f(x) = cos x in [a,b].

13. Find the values of λ and μ for which the system $2x + 5y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ has no solution.

14. Check whether the matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal or not. Also find A^{-1}, if it exists.

15. Solve the system of equations $x + y + z = 4$, $x - y + z = 0$, $2x + y + z = 5$ using Cramer's rule.

PART – C

Answer **any 4** questions from among the questions 16 to 22. **Each** question carries **3** marks.

16. If y = a cos log x + b sin log x, show that $x^2y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$.

17. Find the n^{th} derivative of $y = x\log\frac{x-1}{x+1}$.

18. Show that $x - \frac{x^3}{6}\sin x < x - \frac{x^3}{6} - \frac{x^5}{120}$, if $x > 0$.

19. Find a, b, c so that $\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin^2 x} = 2$.

20. Using Gauss-Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

21. Are the vectors (2,1, 1), (2, 0, –1), (4, 2, 1) linearly independent ? If so find the relation between them.

22. Find the radius of convergence of the curve, $y = c\cosh\left(\frac{x}{c}\right)$ at (0, c).

PART – D

Answer **any 2** questions from among the questions 23 to 26. **Each** question carries **5** marks.

23. a) If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. Also find the value of y_n when x = 0.

    b) Find the n^{th} derivative of $y = e^{5x}\cos x \cos 3x$.

24. a) Prove that $\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \ldots$

    b) Evaluate $\lim\limits_{x\to 0} \frac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$.

25. a) Find two non-singular matrices P and Q such that PAQ is in normal form, where $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

    b) Using partition method, find the inverse of $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$.

26. a) Find the centre of curvature of the curve $y^2 = 4ax$ at $(at^2, 2at)$.

    b) Write the spherical equation and cylindrical equation of $z = \sqrt{x^2 + y^2}$.