K16U 2251

Reg. No. : ..............................

Name : ..............................

**III Semester B.Sc. Degree (CCSS-Supple./Improve.) Examination, November 2016**
**COMPLEMENTARY COURSE IN MATHEMATICS**
**3C 03 MAT : Differential Equations, Laplace Transforms, Fourier Series and Partial Differential Equations**
**(2013 and Earlier Admissions)**

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Weightage : 30

1. Fill in the blanks :

a) Laplace transform of f(t) = 1 is ___________

b) Laplace transform of f(t) = sin 2t is ___________

c) Inverse Laplace transform of $\frac{1}{s+a}$ is ___________

d) Inverse Laplace transform of $\frac{1}{s^3}$ is ___________ **(Weightage : 1)**

Answer **any six** from the following. (Weightage **1 each**).

2. What do you mean by a first order linear differential equation ? Give an example.

3. Solve the differential equation $9y\frac{dy}{dx} + 4x = 0$.

4. Solve $(x^3 + 3xy^2)\,dx + (3x^2y + y^3)\,dy = 0$.

5. Find Laplace transform of $\cos^3 2t$.

**P.T.O.**

(5) 6. Find inverse Laplace transform of $\frac{2s+5}{s^2+4s-5}$.

7. What do you mean by an even function ? Give an example.

8. Write Euler's formula for Fourier series expansion of functions with period $2\pi$.

9. Show that $u = \cos 4t \sin 2x$ is a solution to the one dimensional wave equation with suitable c.

10. Solve the partial differential equation $u_{xx} - u = 0$. **(Weightage 6×1=6)**

Answer **any seven** from the following. (Weightage **2 each**).

11. Solve the initial value problem $y'' + y' - 2y = 0$, $y(0) = 4$, $y'(0) = -5$.

12. Solve $x^3 y''' + 3x^2 y'' + xy' + y = 0$.

13. Prove that the family of parabolas $x^2 = 4a(y + a)$ is self orthogonal.

14. Define unit step function. Also find its Laplace transform.

15. Find the Laplace transform of periodic function of period T, defined by

$$f(t) = \frac{kt}{T},\ 0 < t < T.$$

16. Using convolution theorem, find inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$.

17. Obtain the Fourier series expansion of $f(x) = |x|$ in the interval $(-\pi, \pi)$.

18. Obtain the half range Fourier cosine series expansion of $f(x) = x^2$ in $0 < x < \pi$.

19. Using separation of variables, solve $u_{xx} + u_{yy} = 0$.

20. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$$

**(Weightage 7×2=14)**

Answer **any three** from the following. (Weightage **3 each**).

21. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = \sin 3x + x^2$.

22. Using method of variation of parameters, solve $y'' + 4y = \sec 2x$.

23. Using Laplace transform, solve the initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t, \quad y(0) = 0, \; y'(0) = 1.$$

24. Obtain the Fourier series expansion of

$$f(x) = \begin{cases} x & \text{if } 0 < x < \pi \\ \pi - x & \text{if } \pi < x < 2\pi \end{cases} \quad \text{and } f(x + 2\pi) = f(x).$$

25. Derive D'Alembert's solution to one dimensional wave equation.

**(Weightage 3×3=9)**

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