K15U 0142

Reg. No. : VII

Name : ....

III Semester B.Sc./B.C.A. Degree (CCSS – Supple./Imp.) Examination, November 2015
COMPLEMENTARY COURSE IN MATHEMATICS
3C03 MAT : Differential Equations, Laplace Transforms, Fourier Series and Partial Differential Equations
(2013 and Earlier Admission)

Time: 3 Hours

Max. Weightage: 30

1. Fill in the blanks :

   a) Degree of the differential equation $\dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 = \sin x$ is _____

   b) Laplace transform of $\cos at$ is _____

   c) If $f(-x) = f(x)$, the function is said to be _____

   d) Two dimensional Laplace equation is _____   **(Weightage : 1)**

(Answer **any six** from the following) :

2. Give an example for a Bernoulli's ordinary differential equation.

3. Solve $\dfrac{dy}{dx} = 1 + y^2$.

4. What do you mean by Wronskian of two functions y_1 and y_2 ?

5. Explain the procedure to find the orthogonal trajectory of a given curve.

6. State convolution theorem for Laplace transform.

7. Find Laplace transform of $\cos^2 2t$.

8. Find inverse Laplace transform of $\dfrac{1}{s^2 + 2s + 5}$.

9. State half range Fourier Sine series formula.

10. Verify that $u = x^2 + t^2$ satisfies one dimensional wave equation by assuming suitable value for the constant in the heat equation.   **(Weightage : 6×1=6)**

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(Answer **any seven** from the following) :

11. Solve $(2x - 4y + 5)y' + x - 2y + 3 = 0$.

12. Find the orthogonal trajectories of $y^2 = 4ax$.

13. Using method of variation of parameters, solve $y'' + 7y' - 8y = e^{2x}$.

14. Solve $x^2y'' + xy' + y = \sin(\log x^2)$.

15. Find the Laplace transform of $\dfrac{1 - e^t}{t}$.

16. Find inverse Laplace transform of $\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$.

17. Find the Laplace transform of the periodic function $f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases}$

    $f(t + 2a) = f(t)$.

18. Find the Fourier cosine series of $f(x) = x^2$ in $(0, \pi)$.

19. Find a solution u(x, y) of the partial differential equation $u_{xy} + u_x = 0$.

20. Using the method of separation of variables, solve the PDE $u_{xx} - u = 0$.

    (Weightage : 7×2=14)

(Answer **any three** from the following) :

21. Solve the initial value problem $y'' + 2y' + y = e^{-x}$, $y(0) = -1$, $y'(0) = 1$.

22. Using Laplace transform, solve $y''' - 3y'' + 3y' - y = t^2 e^t$, $y(0) = 1$, $y'(0) = 0$ and
    $y''(0) = -2$.

23. Find the Fourier series of $f(x) = x^2$, $-\pi < x < \pi$, given that f(x) is periodic with

    period 2π. Also deduce that $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dots = \dfrac{\pi^2}{6}$.

24. Expand $f(x) = |x|$ in Fourier series in the interval $(-l, l)$.

25. Using the method of separation of variables, obtain the possible solution of one
    dimensional wave equation.

    (Weightage : 3×3=9)