K16U 1105

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2016
COMPLEMENTARY COURSE IN MATHEMATICS
2C02 MAT : Differential and Integral Calculus
(2013 and Earlier Admn.)

Time : 3 Hours   Max. Weightage : 30

1. Fill in the blanks :

   a) $\frac{d}{dx}(\cosh^{-1}x) =$ ____________

   b) The derivative of $(\log x)^{\log x}$ at $x = e$ is equal to ____________

   c) $\lim\limits_{x \to 0} \frac{\sin x}{x} =$ ____________

   d) Determine the limit of $\frac{1 + \log x - x}{1 - 2x + x^2} (x \to 1)$.   **(Weightage 1)**

Answer **any six** from the following (weightage **1 each**) :

2. Determine the limit of $\frac{e^x - e^{\sin x}}{x - \sin x} (x \to 0)$.

3. Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$.

4. State Taylor's theorem.

5. Verify Rolle's theorem for $f(x) = x(x + 3)\, e^{\frac{-x}{2}}$ in [–3, 0].

6. Prove that the radius of curvature of the catenary $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$ is $\frac{y^2}{a}$.

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7. If $z = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$. Prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2-y^2}{x^2+y^2}$.

8. Find the area of the loop of the curve $r^2 = a^2\cos\theta$.

9. Find the surface of the solid formed by revolving the cardiod $r = a(1 + \cos\theta)$ about the initial line.

10. Evaluate $\int_1^{\log 8}\int_0^{\log y} e^{x+y}\, dx\, dy$. **(Weightage 6×1=6)**

Answer **any seven** from the following (weightage **2 each**) :

11. Evaluate $\iint r^2 \sin\theta \, dr\, d\theta$ over the area of the cardiod $r = a(1 - \cos\theta)$ above the initial line.

12. The area included between the curve $y^2 = x^3$ and $x^2 = y^3$ is rotated about the x-axis. Find the volume of the solid generated.

13. Expand cosx by Maclaurin's series.

14. If $u = (1 - 2xy + y^2)^{-\frac{1}{2}}$ show that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$.

15. Find the surface generated by the revolution of an arc of the centenary $y = c\cos\frac{x}{c}$ about the axis of x.

16. Integrate : (i) $\sin^3 x\, dx$, ii) $\sin^4 x dx$.

17. If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$, prove that $\frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$.

18. Find the area in the first quadrant bounded by the x-axis and the curve $x^2 + y^2 = 10$, $y^2 = 9x$.

19. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$.

20. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$. **(Weightage 7×2=14)**

Answer **any three** from the following (weightage **3 each**) :

21. Find the volume bounded by the paraboloid $x^2 + y^2 = 16 - z$, the cylinder $x^2 + y^2 = 4$ and the plane $z = 0$.

22. Expand $e^{a\sin^{-1}x}$ in powers of x by Maclaurin's theorem and hence obtain the value of e^{θ}.

23. Show that the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $(ax)^{\frac{2}{3}} - (by)^{\frac{2}{3}} = (a^2 + b^2)^{\frac{2}{3}}$.

24. If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr}$.

25. Show by double integration that the area lying inside the circle $r = a\sin\theta$ and outside the cardiod $r = a(1 - \cos\theta)$ is $\frac{a^2}{4}(4 - \pi)$. **(Weightage 3×3=9)**