K17U 1119

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (C.C.S.S. – Supple.) Examination, May 2017
COMPLEMENTARY COURSE IN MATHEMATICS
2C02 MAT : Differential and Integral Calculus
(2009-13 Admns.)

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

a) $\frac{d}{dx}(\text{sech}x) =$ __________

b) The differential coefficient of log tanx is __________

c) The differential coefficient of f(log x) where f(x) = log x is __________

d) If $x = a(1 - \cos\theta)$, $y = a(\theta - \sin\theta)$, then $\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}$ is __________ **(Weightage 1)**

Answer **any six** from the following (weightage **1 each**) :

2. Determine the limit of $\frac{\log(x-a)}{\log\left(e^x - e^0\right)}$ as $x \to a$.

3. Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$.

4. State Maclaurin's theorem.

5. The function f(x) is defined in [0, 1] as follows $f(x) = 1$ for $0 < x < \frac{1}{2}$.

$= 2$ for $\frac{1}{2} \leq x \leq 1$

Show that f(x) satisfies none of the conditions of Rolle's theorem, yet $f'(x) = 0$ for many points in [0, 1].

P.T.O.

6. Find the radius of curvature of a curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ using equation in parametric coordinates.

7. State and prove the Euler's theorem on homogeneous functions.

8. Find the area of the cardiod $r = a(1 - \cos\theta)$.

9. Find the whole length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

10. $\int_{1}^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y\,dx\,dy$. **(Weightage : 6×1=6)**

Answer **any seven** from the following (weightage **2 each**).

11. Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dx\,dy$ and hence evaluate the same.

12. The area included between the curve $y^2 = x^3$ and $x^2 = y^3$ is rotated about the x-axis. Find the volume of the solid generated.

13. Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$.

14. If $u = \log(\tan x + \tan y + \tan z)$. Show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$

15. Find the surface of the solid formed by revolving the cardiod $r = a(1 + \cos\theta)$ about the initial line.

16. Find the reduction formula for $\int \sin^n x\,dx$.

17. If $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$, $f'(0) = a$ and $f(0) = b$, find $f''(x)$ where y is independent of x.

18. Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

19. If $y = e^{\tan^{-1}x}$, prove that $(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n + 1)y_n = 0$.

20. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$, **(Weightage : 2×7=14)**

Answer **any three** from the following (weightage **3 each**).

21. Find the volume bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

22. Show if $y = \sin(m \sin^{-1}x)$, then $(1 - x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+m^2y = 0$. Hence expand $\sin m\theta$ in powers of $\sin\theta$.

23. Show that the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=\left(a^2-b^2\right)^{\frac{2}{3}}$.

24. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x=y+z)^2}$.

25. Find by double integration the area of the region enclosed by the curves $x^2 + y^2 = a^2$, $x + y = a$ in the first quadrant. **(Weightage : 3×3=9)**
