K17U 2546

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, November 2017
(2014 Admn. Onwards)
COMPLEMENTARY COURSE IN MATHEMATICS
1C01 MAT-CS : Mathematics for Computer Science I

Time : 3 Hours

Max. Marks : 40

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Find $\frac{dy}{dx}$ when $x = t^3$ and $y = t^2 - t$.
2. Find the derivative of $ln(\sinh x^4)$.
3. If $z = x/y$, find $\frac{\partial z}{\partial y}$.
4. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates. **(1×4=4)**

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2** marks **each**.

5. If $y = x^2\cos x$, show that $x^2y_2 - 4xy + (x^2 + 6)y = 0$.
6. Find the n^{th} derivative of $y = \frac{x+1}{x^2-4}$.
7. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that $x^2y_{n+2} + (2n + 1)\,xy_{n+1} + 2n^2y_n = 0$.
8. Find the 1027^{th} derivative of $g(x) = \cos x$.
9. Verify Rolle's theorem for $f(x) = \log(x^2 + 2) - \log 3$ on $[-1, 1]$.

P.T.O.

10. Evaluate $\lim_{x \to \pi} \frac{x \cos x + \pi}{\sin x}$.

11. If the sides and angles of a plane triangle ABC vary in such a way that its circum radius remains constant, prove that, $\frac{\delta a}{\cos A} + \frac{\delta b}{\cos B} + \frac{\delta c}{\cos C} = 0$, where δa, δb and δc denote small increments in the sides a, b and c respectively.

12. Verify Euler's theorem when $f(x, y) = ax^2 + 2hxy + by^2$.

13. Find the radius of curvature at any point (x, y) of the curve, $y = a \log \sec(x/a)$.

(2×7=14)

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Prove that $f\left(\frac{x^2}{1+x}\right) = f(x) - \frac{x}{1+x} f'(x) + \frac{x^2}{(1+x)^2} \frac{f''(x)}{2!} + \ldots$

15. Evaluate $\lim_{x \to \pi/4} (\tan x)^{\tan 2x}$.

16. Verify Lagrange's mean value theorem for the function $f(x) = (x - 4)(x - 6)(x - 8)$ in [4, 10].

17. If $u = 3(lx + my + nz)^2 - (x^2 + y^2 + z^2)$ and $l^2 + m^2 + n^2 = 1$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

18. Show that the chord of curvature through the pole of the equiangular spiral $r = ae^{\theta \cot \alpha}$ is 2r.

19. What do the following equations represent in three dimensional geometry ?

a) $xyz = 0$ in Cartesian coordinates.

b) $\rho = 0$ in spherical coordinates.

c) $\phi = 0$ in spherical coordinates.

(3×4=12)

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each.**

20. Use Maclaurin's theorem to find the expansion of log $(1+e^x)$ in ascending powers of x to the term containing x^4.

21. Find the intervals in which the function given by

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11 \text{ is}$$

a) strictly increasing

b) strictly decreasing.

22. Find the centre of curvature of the four cusped hypocycloid, $x = a\cos^3\theta$, $y = a\sin^3\theta$.

23. a) Convert the point $(1, -1, -\sqrt{2})$ from Cartesian to spherical coordinates.

    b) Find an equation in spherical coordinates for the surface $3x^2 - x + 3y^2 + 3z^2 = 0$.

(5×2=10)