K25U 0831

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2023 Admissions)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT-PH : Mathematics for Physics – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Write one dimensional wave equation.
2. State fundamental theorem on superposition.
3. Define gradient field.
4. Define smooth surface.
5. State geometrical significance of trapezoidal rule.

PART – B

Answer **any seven** questions. **Each** question carries **2** marks. **(7×2=14)**

6. Verify that $u = e^{-t} \sin x$ is a solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$.
7. Solve $u_{xx} - u = 0$.
8. Find the type of the PDE $u_{xx} + 4u_{yy} = 0$.
9. Evaluate $\int_C (x + y)\, dS$ where C is the straight line segment x = t, y = (1 – t), z = 0 from (0, 1, 0) to (1, 0, 0).

P.T.O.

10. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the xy plane.

11. Find the gradient of $f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$.

12. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9$, $0 \leq z \leq 5$.

13. If $r(u, v) = \cos u \cos v i + \cos u \sin v j + uk$, find $|r_u \times r_v|$.

14. State divergence theorem.

15. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule taking $h = 0.25$.

16. Write the fourth order Runge Kutta formula.

PART – C

Answer **any four** questions. **Each** question carries **three** marks. **(4×3=12)**

17. Find solutions $u = u(x, y)$ of the PDE $u_{xy} = - u_x$.

18. Find the work done by the force field $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ along the curve $r(t) = ti + t^2j + t^3k$, $0 \leq t \leq 1$ from (0, 0, 0) to (1, 1, 1).

19. Show that $F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative over its natural domain and find a potential function for it.

20. Find the surface area of the hemisphere of radius a.

21. Find the curl of $F = (x^2 - z)i + xe^zj + xyk$.

22. Given $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$, obtain the Taylor series for y(x) and compute y(0.1) correct to four decimal places.

23. Explain Euler's method.

PART – D

Answer **any two** questions. **Each** question carries **five** marks. **(2×5=10)**

24. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ satisfying u (0, t) = 0 and u(L, t) = 0 for all t $\geq$ 0 and corresponding to the triangular initial deflection, $f(x) = \begin{cases} \frac{2k}{L}x, & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x), & \text{if } \frac{L}{2} < x < L \end{cases}$ and initial velocity zero.

25. State normal form of Green's theorem. Verify this theorem for the vector field F = yi – xj and the region R is bounded by the circle C : r(t) = a cos ti + a sin tj, $0 \leq t \leq 2\pi$.

26. State Stoke's theorem. Verify this theorem for the vector field F = yi – xj over the hemisphere S : $x^2 + y^2 + z^2 = 9$, $z \geq 0$, bounded by the circle C : $x^2 + y^2 = 9$, z = 0.

27. Use Runge-Kutta fourth order formula to find y(0.2) and y(0.4) given that $y' = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$